Modelling Stock Prices and Exchange Rates during Turbulent Periods: New Insights from Multivariate GARCH Parameterizations

Sagheer Muhammad[1] Ghulam Ali Bhatti [2] Abdul Raheman[3]

Abstract

This study investigates the dynamic conditional correlation structure and volatility transmission between foreign exchange and stock markets. For this purpose, the study uses the dataset from July 1st, 2001 to June 30th, 2019, as an entire sample period to generalize the interaction between two vital segments of the economy. This study employs a range of econometric specifications like the Johansen and Juselius (1992) co-integration test to check the long-term drift components, DCC-GARCH of Engle (2002), for dynamic correlation structure and bivariate BEKK for conditional volatility transmission. The empirical findings of Johansen and Juselius (1992) co-integration suggest the long-run linkage between the economic variables. The results of the BEKK-GARCH framework suggest that shock and volatility observed in the foreign exchange market significantly affect the Pakistani stock market. The uni-directionality found in the underlying linkage is aligned with the stock-oriented model, suggesting causality flow from the foreign exchange market to the equity market. Further, the results of the DCC-GARCH framework indicates the persistence of ARCH and GARCH effect on overall sample subsets. Wherein, the mean-reverting process of conditional volatility series for foreign exchange market observed the permanent shock, which does not die away with the passage of time. This framework also revealed the time-varying nature of volatility structure and observed that correlation structure behaves asymmetrically in different sample subsets. In the progress of time, the magnitude of this association, especially in crisis and post crisis periods, remained higher as compared to the pre-crisis period. The empirical findings of this study are essential for practitioners involved in policymaking and for managers seeking hedging decisions for the optimum allocation of the resources.

Keywords: Stock prices, Exchange rates, Shock Dependence, Volatility Transmission, BEKK-GARCH, DCC-GARCH
JEL Classifications: *C32, F31, G15*

Introduction

Classical financial theories state that both the exchange rate and equity prices are vital macro-economic factors that are fundamentally intertwined. Movements in the exchange rate affect stock price assuming that depreciation (appreciation) in the local currency market increases (decreases) the competitiveness of domestic firms in the

[1] University of Gujrat, E-mail: sagheer.muhammad@hotmail.com
[2] University of Sargodha
[3] Quaid-i-Azam University

international market and their future earnings, trade balances are affected through the change in country output and resultantly, increasing (decreasing) local equity prices (Dornbusch & Fischer, 1980). Contrarily, change in equity prices affect exchange rates, since movement in equity prices influence the demand and supply curve of money. An upward shift in equity prices leads to an adjustment in currency rates, and local and international portfolio of financial assets is changed due to the sale and purchase of money in the global market. Over the last three decades, the linkage between foreign currency rates and equity prices is at the heart of attraction for theoretical and empirical research. It is probably due to global financial liberalization that removed the barriers of capital flow, application of dynamic exchange rate regime, advancements in world economic and financial integration, and portfolio diversifications at the international level. Therefore, it is vital to have a clear understanding of how shocks from external sources such as financial, economic or political disturbances affect the interlink-age between foreign exchange rates and equity market returns.

The purpose of this study stems from investigating the linearity and nonlinearity association between the vital economic indicators over the different states of the economy. To find the linearity in the relationship, we employ the vector auto-regression (VAR) technique, whereas the non-linear relationship is investigated by using the multivariate-GARCH frameworks. The specification was introduced by Engle (2002) as a DCC model. This paper subsidizes to the extant literature in many bends significantly. In contrast, much historical research has been done in investigating the time-varying interaction between stock and foreign currency markets by employing both linear and non-linear techniques on developed and few emerging markets (e.g., Pan, Fok, & Liu, 2007; Ramasamy & Yeung, 2005; Zhao, 2010) but minimal work has been done on developing countries like Pakistan. Secondly, this paper also extends the analysis between equity prices and currency rates to a non-linear relationship. This is due to the theoretical underpinning of financial variables, which often predict the non-linear association between them (Zhao, 2010).

This study contributes to literature in many ways. Firstly, the empirical literature on this very nature of the relationship is still in a state of flex. The lack of practical and theoretical consensus is required by examining these two vital indicators of the economy. This attempt has been made to investigate the interlink-age on a broad spectrum of data using different flexible specifications, synthesizing nature into a more thorough orientation for the practitioners and policymakers. For this purpose, a set of linear and non-linear models have been employed to model the multiple aspects of the mean-variance structure of this vital relationship. In order to provide a more detailed understanding of the nature of volatility transmission flow, shock dependence and correlation structure across the different subsets, this study has uniquely employed dynamic and flexible parameterization, namely, BEKK-GARCH and DCC-GARCH. These employed models can estimate the complex variance-covariance system by imposing the restrictions of semi-positive definiteness on coefficient matrices.

Secondly, while many efforts have been made in exploring the returns-volatility transmission mechanism between these financial assets in both developed and some emerging markets (e.g., Chiang & Yang, 2003; Granger et al., 2000; Pan, Fok, & Liu, 2007; Ramasamy & Yeung, 2005; Wu, 2000; Zhao, 2010) very limited work has been done on emerging markets like Pakistan. Thirdly, the present study extends the analysis by using more flexible volatility transmission parameterization to model the currency rates and equity prices. The parameterizations are capable of predicting the variance-covariance matrix in a time-varying way. The prime reasons to employ such specifications stem from the empirical evidence that the volatilities of both the stock market and exchange rates tend to move together over a period of time. So, for shock and volatility transmission, we employed the Bivariate-BEKK model, and for correlation structure, the DCC-GARCH model of Engle (2002) has been used.

Further, this study also analyzes the financial crisis period. For this purpose, the study period is segregated into three sample subsets, namely, pre-crisis, crisis and post crisis periods. Empirical and theoretical literature reflects that shocks from a financial crisis could also influence the relationship between exchange rate and stock market returns (Amaefula & Asare, 2013; Chan, Gup & Pan, 1997; Kganyago & Gumbo, 2015). However, this has attracted very little attention from most existing studies which focus more on the immediate effects (or additive) models (Ouma & Muriu, 2014; Kirui, Wawire & Onono, 2014; Kumar & Puja, 2012; Razzaque & Olga, 2013).

The empirical findings of the VAR model suggest that equity prices and currency rates are empirically interlinked through long-run drifts over the entire data period undertaken. These empirical findings are consistent with the study of Luqman and Kousar (2018), who used high-frequency data to scrutinize the vital economic variables.

The results of the BEKK-GARCH framework suggest that shock and volatility observed in the foreign exchange market significantly affect the Pakistani stock market. The uni-directionality found in the underlying linkage is consistent with the stock oriented model, suggesting causality flow from the foreign exchange market to the equity market. Further, the results of the DCC-GARCH framework indicates the persistence of ARCH and GARCH effect on overall sample subsets. Wherein, the mean-reverting process of conditional volatility series for foreign exchange market observed the permanent shock, which does not die away with the passage of time. This framework also reveals the time-varying nature of volatility structure and observes that correlation structure behaves asymmetrically in different sample subsets. With the passage of time, this association's magnitude, especially in crisis and post crisis periods remains higher compared to the pre-crisis period.

Literature Review

Theoretically, the intertwining between foreign currency rates and stock prices is explained by two theoretical models. The first model is the "flow-oriented" model presented by Durnbusch and Fischer (1980), and the second model is the "stock-oriented" model presented by Branson (1983) and Frankel (1983). Theoretically, the Flow-oriented model depicts that the current account balance of an economy generally determines exchange rates. The level of real income and output of an economy is influenced since that change happened. In the meantime, exchange rates are also affected by movements in stock prices by changing aggregate demand for money. For example, a fall in stock price directly decreases the investors' wealth, causing a reduction of liquidity in the economy. On the contrary, a low degree of liquidity decreases capital outflow and interest rates that depreciate the currency. Hence, stock price fluctuations influence different economy's macroeconomic factors such as wealth, interest rates, and liquidity. Primarily, large bodies of empirical literature have explained the very nature of the relationship between exchange rate and equity prices by applying correlation analysis, vector autoregressive techniques such as causality and co-integration. Aggarwal (1981) finds a positive association between real currency rates and equity prices for the US and UK markets. However, a strong negative correlation and statistically significant relationship are postulated by Soenen and Hennigar (1988) between the US equity prices and foreign currency rates from 1980 to 1986. However, Bartov and Bodnar (1994) couldn't find a significant correlation between the underlying economic variables using data from the US firms.

Numerous historical studies applied causality and co-integration analysis to investigate the interaction between foreign exchange rates and equity prices. Some studies were documented by confirming co-integration, whereas others were unable to display any long-run co-movement between both financial markets. The empirical works using causality techniques in their analysis could not find consistent results; however, findings have been changed across the countries. Some of them reported unidirectional spillover moved from equity prices to foreign currency rates. Some other suggested bidirectional causality between these two vital indicators and emphasized that both the markets were independent and were not associated with each other. Smyth and Nandha (2003) examined the association for Pakistan, Bangladesh, Sri Lanka, and India using VAR methodology and failed to find long term inter-linkage between all sample countries. Anyhow, the result from the Granger causality test was mingled, and the direction of causality could not be established. Zhao (2010) studied dynamic interaction between foreign currency prices and equity by applying VAR and multivariate GARCH (MGARCH) techniques in China and revealed that both variables had no long-run equilibrium. Hence, the multivariate GARCH technique exhibits a time-varying relationship. This study also found a time-varying causality between the said variables. Adjasi et al. (2011) analyzed the interlocking in seven African countries, and no similarity is established in results regarding short- and long-run linkage.

The review of the literature indicates that previous studies failed to develop a consensus about inter-linkage between foreign currency and stock markets using a range of econometric specifications. Though, the very nature of the interaction is still in a state of flux since the current study indents to explore the non-linear inter-linkage between foreign currency rates and equity prices using the parameterization of Multivariate GARCH specifications.

Data and Methodology

The study uses the KSE-100 index, the premier index of Pakistan, along with the US/Dollar exchange rates. The equity market data has been collected from the website of yahoo finance. Data related to foreign currency rates is taken from the annual report of the State Bank of Pakistan. All the collected information is at the weekly frequency covering a period from July 1st, 2001 to June 30th, 2019, aggregating 938 values. The data set has been further divided into three parts to understand the relationship in Global Financial crisis. The study also investigates the interaction over post and pre-crisis to study how business and relationships have changed in the aftermath. Regulators worldwide, including Pakistan, initiated significant steps to protect both the stock and foreign exchange market to collapse further.

To overcome the issue of non-stationarity, continuously compounded returns have been computed. For stock price and foreign exchange returns calculation following formula has been used:

$$R_t = ln\left[\frac{P_t}{P_{t-1}}\right]$$

Where P_t represent the current prices of the KSE-100 index and PKR/US Dollar, and P_{t-1} represents the lagged or previous day prices of the markets. In order to investigate the dynamic interlinkage, this study employs a range of econometric specifications like the Johansen and Juselius (1992) co-integration test to check the long-term drift components, DCC-GARCH of Engle (2002), for dynamic correlation structure and bivariate BEKK for conditional volatility transmission between the foreign exchange and the equity prices.

BEKK-GARCH (1, 1)

The extension of the univariate-GARCH framework to the multi-series needs to model variance and covariance equations for the respective series. As we aim to estimate the conditional variance-covariance matrix, so in the first step, we define mean and variance equations for both the series in the following manners:

$$r_s = \mu_s + \phi_s r_{s-1} + \varepsilon_s \quad (1)$$

$$r_{exr} = \mu_{exr} + \phi_{exr} r_{exr-1} + \varepsilon_{exr} \quad (2)$$

where r_s and r_{exr} denote the return series for equity prices and exchange rates, representing the autoregressive models as their values conditional upon to their respective lagged series values r_{s-1} and r_{exr-1}. whereas μ_s and μ_{exr} represent parameters of long term alpha with the unexplained portion of the model ε_s and ε_{exr} for equity and foreign exchange markets.

For the estimation of the conditional variance-covariance matrix, the bivariate BEKK framework of Engle and Kroner (1995) is used. The unrestricted-BEKK parametrization can be explained through matrix notion, which is described as:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \quad (3)$$

where the elements of the individual matrix for C, A and B are given as under:

$$A = \begin{bmatrix} \beta_{exr,t} & \beta_{exrs,t} \\ \beta_{sexr,t} & \beta_{s,t} \end{bmatrix} \qquad B = \begin{bmatrix} \delta_{exr,t} & \delta_{exrs,t} \\ \delta_{sexr,t} & \delta_{s,t} \end{bmatrix} \qquad C = \begin{bmatrix} \alpha_{exr,t} & \alpha_{sexr,t} \\ 0 & \alpha_{s,t} \end{bmatrix}$$

Where C is an upper triangular matrix of constants, A is a (2x2) square matrix and represents the conditional volatility function for currency rates and equity prices and the elements of the matrix estimate the sensitivity of conditional variance in response to previous price behaviour. The (2x2) square matrix B represents the transmission effects to the degree at which today's volatility is determined by the historical period volatility between both the series.

Modelling equation 3 in bivariate BEKK-GARCH setting gives the following functions indicating the individual and joint series dependence separately:

$$h^2_{exr} = c_{exr} + \beta^2_{exr}\varepsilon^2_{exr-1} + 2\beta_{exr}\beta_{sexr}\varepsilon_{s-1}\varepsilon_{exr-1} + \beta^2_{sexr}\varepsilon^2_{s-1} + \delta^2_{exr}h^2_{exr-1} + 2\delta_{exr}\delta_{sexr}h_{sexr-1} + \delta^2_{sexr}h^2_{s-1} \quad (4)$$

$$h_{sexr} = c_{sexr} + \beta_s\beta_{exr}\varepsilon^2_{s-1} + (\beta_{exrs}\beta_{sexr} + \beta_s\beta_{exr})\varepsilon_{s-1}\varepsilon_{exr-1} + \beta_{exrs}\beta_{exr}\varepsilon^2_{exr-1} + \delta_s\delta_{exr}h^2_{s-1} + (\delta_{exrs}\delta_{sexr} + \delta_s\delta_{exr})h_{sexr-1} + \delta_{exrs}\delta_{exr}h^2_{s-1} \quad (5)$$

$$h_s^2 = c_s + \beta_{exr}^2 \varepsilon_{exr-1}^2 + 2\beta_{oexr}\beta_{sexr}\varepsilon_{s-1}\varepsilon_{exr-1} + \beta_{sexr}^2\varepsilon_{s-1}^2 + \delta_{exr}^2 h_{exr-1}^2 + 2\delta_{exr}\delta_{sexr}h_{sexr-1} + \delta_{sexr}^2 h_{s-1}^2 \quad (6)$$

The maximum likelihood function under the assumption of restricted normality employs to find the Bivariate-BEKK GARCH parameters estimates, which is described below:

$$Max\ logL_T(\theta) = \sum_{t=1}^{T} l_t(\theta) \quad (7)$$

$$l_t(\theta) = -\frac{TN}{2}log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(log|H_t| + e_t' H_t^{-1} e_t) \quad (8)$$

Where θ presents unknown parameters to be estimated, N indicates the variable series modelled, whereas T represents number quantities.

DCC-GARCH Model

This study employs the DCC-GARCH model to estimate the volatility inter-linkage between stock and currency markets. This framework was proposed by Engle (2002) as a simplified version in a multivariate GARCH family setting. The employed setup utilizes the two-step procedure to observe the shift in the conditional correlation structure of the multiple series across the different economic states.

In the first step, the univariate-GARCH model is used to model the conditional volatility of the stock and currency markets separately. Therefore dynamic conditional second moments of returns on equity i at time t is represented as the following GARCH setup:

$$h_{i,t} = \omega_i + \sum_{j=1}^{p} \alpha_{ij}\varepsilon_{i,t-j}^2 + \sum_{j=1}^{q} \beta_{ij} h_{i,t-j} \quad (9)$$

Where $\varepsilon_t = \sqrt[\eta]{h_{it}}$ *is the residual term considered as conditionally multivariate normal*

ω_i *= Constant Term*

α_{ij} *= ARCH Effect coefficient measuring the shock dependence*

β_{ij} *=GARCH Effect parameter measuring the volatility persistence*

$h_{i,t-j}$ *= Lagged conditional correlation matrix*

In the second step, the standardized output of error terms from the first stage is then used to estimate the dynamic correlation matrix, which is described as a diagonal matrix stated below:

$$r_{i,t}|\Psi_{t-1} \sim N(0, H_t),\ H_t = D_t \Gamma_t D_t \quad (10)$$

$$\varepsilon_t = D_t \eta_{it},\ E_{t-1}(\varepsilon_t, \varepsilon_t') = \Gamma_t$$

where H_t represents a positive definite conditional covariance matrix, $r_{i,t}$ is normally distributed returns with zero mean and unit variance, Ψ_{t-1} indicates information set available at t-1, and D_t is a k x k diagonal matrix of time-varying residuals obtained from step 1 with $\sqrt{h_{i,t}}$ on the i^{th} diagonal,

$$D_t = \begin{pmatrix} \sqrt{h_{11,t}} & 0 & \cdots & 0 \\ 0 & \sqrt{h_{22,t}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sqrt{h_{mn,t}} \end{pmatrix}$$

Results and Discussions

Table-1 shows the results of descriptive statistics calculated using the natural log difference of the KSE-100 index and foreign currency rates for the entire sample period under study. The mean weekly returns in percentage for the benchmark index (KSE-100) is 0.34% and in annualized equal to 17.19%. In contrast, the weekly mean returns for PKR/US dollar foreign exchange rate is negative -0.0058 %, which is about 0.302 % loss annually.

Table 1: *Descriptive Statistics*

	Returns, KSE-100	Returns, Exchange Rates
Mean	.0034	-.0000576
Median	.0061	-.000084
Maximum	.1091	.07208
Minimum	-.2009	-.05181
Standard Deviation	.0308	.0072
Skewness	-1.1407	2.1591
Kurtosis	8.3626	31.9037
Jarque–Bera test	1327.393	33380.01
Probability	.000	.000
Observations	938	938

The maximum weekly return for the Pakistan stock market under the study period is 10.91%, whereas the weekly loss occurred during the period is 20.09%. Similarly, the maximum weekly return for the PKR/US Dollar Exchange rates is 7.20%, while the weekly loss occurred during the period is 5.18%. The rate of volatility for the series has been investigated by the standard deviation, which is 3.08% and 0.72% for Pakistan Stock Exchange and Foreign Exchange Market, respectively. Stats indicate that the Pakistan Stock Exchange is more volatile than the PKR/US Dollar exchange rates. The percentage deviations in foreign currency rates fall between -5.18% to 7.20%. Convincingly, volatility clustering is significantly evident in both the economic series under the data period. Besides, the values of kurtosis indicated that each of the change series is leptokurtic.

Unit Root Test

To establish the stationarity of the series, Table 2 presents the unit root test performed both at the level and the first difference using the Augmented Dickey-Fuller and Philip-Perron (PP) tests.

Table 2: *Unit Root Test*

	Stock Prices				Exchange Rates			
	ADF Test		PP Test		ADF Test		PP Test	
	Level	1st Diff.	Level	1st Diff.	Level	1st Diff	Level	1st Diff.
T-Stat	-1.969	-26.599	-1.865	-26.808	-0.843	-28.670	-1.182	-29.045
Prob.	(.612)	(.000)	(.67)	(.000)	(.960)	(.000)	(.912)	(.000)
at 1% c.v.	-3.967	-3.967	-3.967	-3.967	-3.967	-3.967	-3.967	-3.967
at 5% c.v.	-3.414	-3.414	-3.414	-3.414	-3.414	-3.414	-3.414	-3.414
at 10% c.v.	-3.129	-3.129	-3.129	-3.129	-3.129	-3.129	-3.129	-3.129

(Author's own calculations)

Note: c.v. denotes the critical values at a given level for both the tests

The results reject the null hypothesis of stationarity at this level. The t-statistic values for both the variables at the level are statistically insignificant, meaning that both series using both the tests are stationery. However, the two series become stationary after indicating that both are of the same integration of order one, I (1). After establishing the level of integration of individual series, the next stage is the test of a long-run relationship using the co-integration technique.

Cointegration Test

Table 3 presents Johansen and Juselius (1992) co-integration test by employing the maximum likelihood method. For the optimal lag order selection, the Schwarz information criterion (SIC) is used, which recommends lag 1 for both equity prices and currency.

Table 3: *Johansen Co integration Test Results*

Series	Hypothesis		λ Trace Statistics	0.05 Critical Value	λ Max-Eigen Statistics	0.05 Critical Value
	H_0	H_1				
LKSE ↔ LEXR	$r = 0$	$r \geq 1$	15.899	15.494	14.951	14.264
	$r \leq 1$	$r \geq 2$	.9475	3.8414	.947	3.841

Note: Entire Sample Period from July 2001 to June 2019 is taken for analysis

The findings of the above table suggest that there exist a co-integrating relationship between equity and foreign exchange currency markets for the sample under investigation. As the value of trace statistic and max-eigenvalue statistics are more significant than their respective 5% critical values, meaning that the null hypothesis of a no-cointegrating vector is rejected. The findings accept the alternative hypothesis, which suggests that one co-integrating vector exists between these two variables. Our empirical results are consistent with the study of Luqman and Kousar (2018), who used high-frequency data to scrutinize the vital economic variables.

Bivariate BEKK-GARCH(1, 1) Model

Table 4 presents the coefficient estimates of the frameworks for equity and foreign exchange markets. GARCH framework in bivariate setting models the mean and variance series separately, in which both the series are conditional upon their lagged values.

Table 4: *Bivariate BEKK-GARCH(1, 1) Parameter Estimates*

Panel A: Conditional Means Estimates (Kse-100 – Exchange Rates)				
Variables	Coefficients	Standard Error	T-Stat	Significance
μ_{kse}	.003	.001	3.424	.001
ϕ_{kse}	.134	.031	4.226	.000
μ_{xr}	-.001	.001	-3.072	.002
ϕ_{xr}	-.007	.041	-.188	.850
Panel B: Conditional Variance-Covariance Estimates (Kse-100 – Exchange Rates)				
α_s	.005	.001	5.626	.000
α_{sxr}	-.001	.001	-.645	.518
α_{xr}	.001	.001	10.826	.000
β_s	.260	.024	10.586	.000
β_{sxr}	-.002	.003	-.645	.518
β_{xrs}	-.196	.093	-2.098	.035
β_{xr}	.370	.025	14.383	.000
δ_s	.949	.009	99.125	.000
δ_{sxr}	.001	.001	.857	0.391
δ_{xrs}	.068	.024	2.765	.005
δ_{xr}	.940	.005	159.981	.000

(Author own Calculations)

The empirical findings of Panel-A show the parameter estimates of conditional mean equations for both equity prices and exchange rates. The one period past returns of the KSE-100 index is denoted with ϕ_{kse} and the value appears to be statistically significant and means historical security market information explains the future market prices. The Panel-B presents the parameter estimates of variance-covariance matrix, in which conditional ARCH component of the model measures the shock dependence for respective series and denoted with β's. Whereas, GARCH effect measures the volatility persistence of individual series alone and the transmission mechanism between foreign exchange to the stock market and vice versa. For equity prices, both the historical period shock and volatility significantly increase the preceding period conditional volatility.

Similarly, for exchange rates, the lagged price behaviour and historical volatility are also significant and positive, suggesting increase in future volatility of the market.

Regarding shock persistence and volatility modelling, the parameter measure shock from currency rates towards the equity market β_{xrs} is statistically significant and negative. This reveals that shock created in the foreign currency market significantly decrease the temporal changes in the equity market. Whereas, the coefficient of shock in equity market towards the foreign currency market β_{sxr} is statistically insignificant. Meaning that temporal shocks created in equity market do not affect the demand for or supply of currency in the foreign exchange market. As for the volatility persistence coefficients, it is revealed that both the foreign currency rates and equity prices historical volatility increase the current fluctuations of these respective series significantly. Taking the volatility transmission coefficients into consideration, the volatility spillover parameter δ_{xrs}, which represent the transmission channel from the foreign exchange market to the equity market is statistically significant with a positive sign, indicating that volatility created in the foreign currency market significantly increases the volatility of the stock market.Whereas, the volatility spillover coefficient from the stock market to the foreign exchange market δ_{sxr} is statistically insignificant. These empirical findings are realistic for the Pakistani market on theoretical grounds for two reasons.

Overall the results of the shock and volatility transmission revealed unidirectionality for underlying markets. The transmission channel flows from the foreign currency market to the equity market. These empirical results are aligned with historical literature suggesting strong transmission between these two vital components of the economy. Finally, it is imperative to state that information set analyzed to create the inference somewhat consists of turbulent periods in which market behaves abnormally, there is a strong belief that some systematic factor caused the irregular transmission between equity and currency markets.

DCC-GARCH (1, 1) Model

This study extends the investigation by further employing the DCC mechanism to scrutinize the nature of correlation structures between equity and forex markets. For this purpose, the unit of analysis has been divided into two parts namely; Univariate-GARCH Estimates which also explain the mean-reverting process of conditional volatility for both the respective series and the estimates of Dynamic Conditional and Un-Conditional Correlation. The parameters estimates of the Univariate-GARCH Model for the entire sample period and sub-sample periods are reported in Table 5. While the parameters estimates of the conditional (DCC-GARCH) and unconditional correlation for the said periods are reported in Table 6.

Table 5: *Univariate GARCH (1, 1) parameter estimates*

Entire Sample Period (July 2001–June 2019)

Parameters	μ (Mean)	ω	α ARCH Parameters	β GARCH Parameters	Mean-Reverting $\alpha+\beta<1$
Stock Returns	.003 (4.023)	1.65E (.902)	.082 (1.867)	.903 (14.723)	.98
Exchange Rate	-.001 (-1.912)	5.45E (.943)	.135 (.031)	.882 (16.952)	1.01

Pre-Crises Period (July 2001–December 2007)

Parameters	μ (Mean)	ω	α ARCH Parameters	β GARCH Parameters	Mean-Reverting $\alpha+\beta<1$
Stock Returns	.005 (3.106)	.001 (1.330)	.142 (1.865)	.771 (6.286)	.91
Exchange Rate	-1.20E (-.077)	6.02E (1.849)	.535 (.003)	.602 (8.524)	1.13

Crises Period (January 2008–December 2011)

Parameters	μ (Mean)	ω	α ARCH Parameters	β GARCH Parameters	Mean-Reverting $\alpha+\beta<1$
Stock Returns	.003 (1.926)	.001 (3.669)	.151 (1.456)	.611 (1.858)	.76
Exchange Rate	.001 (2.334)	5.42E (3.711)	.820 (2.465)	.329 (2.248)	1.15

Post-Crises Period (January 2012–June 2019)

Parameters	μ (Mean)	ω	α ARCH Parameters	β GARCH Parameters	Mean-Reverting $\alpha+\beta<1$
Stock Returns	.003 (3.616)	9.09E (1.893)	.147 (2.142)	.667069 (4.644166)	0.81
Exchange Rate	-.001 (-2.364)	7.73E (.574)	.106 (1.457)	.914 (14.447)	1.02

Note: $\alpha+\beta<1$ *(Indicates the Stationery of value or Mean-Reverting of Conditional Volatility Process)*

In Table 5, parameter estimates of mean and variance equations are presented for both equity and foreign exchange market. Both the equations are modelled separately by using a univariate GARCH (1, 1) framework. The parameter μ represents the lagged returns in the conditional mean equation, whereas ω shows the estimates of constant, while parameter α and β present the estimates of ARCH and GARCH components of the conditional variance equation.

All parameters of mean equation found positive and significant for the stock market, which suggests that today's returns are the function of one-day historical period returns in the entire as well as all sample subsets. Whereas for the foreign exchange market, it appears negative in all the sample subset except in crisis period. The parameters estimates of variance equation α and β validate the employed econometric modelling, presenting ARCH and GARCH effect in both the equity market and foreign currency market. Besides, the joint coefficient $\alpha + \beta$, which reveal the mean-reverting of the conditional volatility process or the stationarity of the value is less than one for the stock market series in an entire as well as all the sample subsets. Whereas, the joint significance of the parameters for the forex market is more significant than one in almost all the sample subsets exhibiting shocks that are above the average level. This phenomenon for the forex market, revealing that increased conditional volatility may have a permanent shock that does not die away. Our empirical results are consistent with the study of King and Wadhwani (1990), which show that an upward change in volatility subsequently increases the size of the transmission effects.

Correlation Structure between Exchange Rates and Stock Prices

Table 6 shows the results of unconditional and conditional correlation of equity prices and currency rates for the sample periods under study.

Table 6: *Results of Unconditional and Dynamic Conditional Correlation (DCC)-GARCH Parameters*

	Coefficient	Entire	Pre-Crises	Crises	Post-Crises
Un-Conditional Correlation	r_{12}	.065** (.045)	.080 (.139)	-.052 (.448)	.073 (.147)
Dynamic Conditional Correlation	δ_1	-.008*** (.000)	-.016*** (.000)	.021 (.691)	-.013*** (.000)
	δ_2	.785*** (.000)	.771*** (.000)	.868*** (.000)	.931*** (.000)
	Log-likelihood	5659.560	2184.255	1228.523	2367.769

Note: Stability condition: Theta(1) + Theta(2) <1 is met for all the above date sets.

**Authors own calculations.*

The findings show that the unconditional correlation between exchange rates and equity prices is significantly positive in the entire sample subset. However, it appears insignificantly positive in both pre and post crises periods and negative in the crisis period. The coefficient of correlation revealing a weak directional relationship between the vital economic variables under study. As for the dynamic conditional correlation, the DCC-GARCH parameters are captured by the magnitude of the shock (δ_1) and the level of persistence (δ_2). The results for unconditional correlation reveal that there exists a significant positive correlation in the entire sample subset. However, the strength of the relationship is too weak, suggesting a weak co-linearity relationship between exchange rates and equity returns. In addition, the coefficient of correlation remains positive and insignificant in pre and post-crisis periods, whereas insignificantly negative in crisis period. The results of DCC-GARCH parameters suggests the time-varying nature of correlation structure in entire as well as all sample subsets. The parameter remains dynamic during the time period covered in this study. The findings for most of the economic states reveal a positive time-varying correlation structure between the underlying variables. The correlation structure during different sample subsets behaves asymmetrically. With the passage of time, the magnitude of this association, especially in crisis and post crisis periods remained higher as compared to the pre-crisis period. In the condition of extreme uncertainty and collapse of the world's economic system, foreign investors immediately withdraw their investments and convert back into local currency (Zubair, 2013; Manasseh et al., 2018).

Conclusion

This paper scrutinized the volatility spillover and dynamic conditional correlation structure between equity and foreign exchange markets. Firstly, to investigate the long term relationship, we applied Johansen and Juselius (1992) technique to find the likelihood of a long-term inter-linkage between the equity and foreign currency markets. Empirical findings of the model suggest that there exist a co-integrating relationship between stock returns and foreign currency rates over the entire sample period. Further to find the volatility spillover and transmission effect, multivariate BEKK-GARCH of Kroner (1995) has been employed for an entire sample period. The empirical findings of multivariate BEKK-GARCH model suggest that shock created in foreign market negatively affect the volatility of the equity market. However, the volatility of the forex market remained unaffected due to a surge in the stock market. These empirical findings are aligned with the flow-oriented model, which explains the transmission channel from currency rates to equity market. In addition, the inference seems entirely logical considering the economic system of Pakistan as the economy is heavily dependent on oil which is the sole ingredient for the manufacturing sector and accounts for the meaningful contribution to GDP. In both the cases of shock persistence and volatility spillover, the transmission channel flows from exchange rates to equity market. Apparently, the interaction is well aligned with the previous literature, which presents strong transmission mechanism between foreign rates and stock exchange. In addition, the conditional correlation between these two vital segments is investigated by using the DCC-GARCH mechanism. The results reveal the positive dynamic interlinkage between the variables across the different states of interval. The correlation structure during different sample subsets behaves asymmetrically. With the passage of time, the magnitude of this association, especially in crisis and post crisis periods remained higher as compared to the pre-crisis period. Our empirical findings are vital for practitioners involved in the policymaking and for managers seeking hedging decisions for optimum allocation of the resources.

References

Adjasi, C. K., Osei, K. A., & Fiawoyife, E. U. (2011). Explaining underpricing of IPOs in frontier markets: Evidence from the Nigeria Stock Exchange. *Research in International Business and Finance, 25(3),* 255-265.

Aggarwal, R. (1981). International differences in capital structure norms: An empirical study of large European companies. *Management International Review, 75-88.*

Amaefula, C. G., & Asare, B. K. (2013). Time Varying Correlation of Stock Returns Relative to Exchange Rate and Inflation Rate and also Their Volatilities: Evidence from Nigeria. *IOSR Journal of Business and Management (IOSR-JBM), 14(1),* 41-49.

Bartov, E., & Bodnar, G. M. (1994). Firm valuation, earnings expectations, and the exchange-rate exposure effect. *The journal of Finance, 49(5),* 1755-1785.

Branson, W. H. (1983). *A model of exchange-rate determination with policy reaction: evidence from monthly data (No. w1135).* National Bureau of Economic Research.

Chan, K. C., Gup, B. E., & Pan, M. S. (1997). International stock market efficiency and integration: A study of eighteen nations. *Journal of business finance & accounting, 24(6),* 803-813.

Chiang, T. C., & Yang, S. Y. (2003). Foreign exchange risk premiums and time-varying equity market risks. *International Journal of Risk Assessment and Management, 4(4),* 310-331.

Dornbusch, R., & Fischer, S. (1980). Exchange rates and the current account. *The American Economic Review, 70(5),* 960-971.

Engle, R. F., & Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH. *Econometric theory, 11(1),* 122-150.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics, 20(3),* 339-350.

Frankel, T. (1983). Fiduciary law. *Cal. L. Rev., 71,* 795.

Granger, C. W., Huangb, B. N., & Yang, C. W. (2000). A bivariate causality between stock prices and exchange rates: evidence from recent Asianflu. *The Quarterly Review of Economics and Finance, 40(3),* 337-354.

Johansen, S., & Juselius, K. (1990). Maximum likelihood estimation and inference on co-integration—with applications to the demand for money. *Oxford Bulletin of Economics and statistics, 52(2),* 169-210.

King, M. A., & Wadhwani, S. (1990). Transmission of volatility between stock markets. The *Review of Financial Studies, 3(1),* 5-33.

Kirui, E., Wawire, N. H., & Onono, P. O. (2014). Macroeconomic variables, volatility and stock market returns: a case of Nairobi securities exchange, Kenya. *International Journal of Economics and Finance, 6(8),* 214-228.

Kganyago, T., & Gumbo, V. (2015). An Empirical Study of the Relationship between Money Market Interest Rates and Stock Market Performance: Evidence from Zimbabwe (2009-2013). *International Journal of Economics and Financial Issues, 5(3),* 638-646.

Kumar, M. (2013). Returns and volatility spillover between stock prices and exchange rates: Empirical evidence from IBSA countries. *International Journal of Emerging Markets, 8(2),* 108-128.

Luqman, R., & Kouser, R. (2018). Asymmetrical Linkages between Foreign Exchange and Stock Markets: Empirical Evidence through Linear and Non-Linear ARDL. *Journal of Risk and Financial Management, 11(3),* 51.

Manasseh, C. O., Chukwu, N. O., Abada, F. C., Ogbuabor, J. E., Onyeka, K. A., & Okoro, O. E. (2019). Interactions between stock prices and exchange rates: An application of multivariate VAR-GARCH model. *Cogent Economics & Finance, 7(1),* 1681573.

Ouma, W. N., & Muriu, P. (2014). The impact of macroeconomic variables on stock market returns in Kenya. *International Journal of Business and Commerce, 3(11),* 1-31.

Pan, M. S., Fok, R. C. W., & Liu, Y. A. (2007). Dynamic linkages between exchange rates and stock prices: Evidence from East Asian markets. *International Review of Economics & Finance, 16(4),* 503-520.

Ramasamy, B., & Yeung, M. C. (2005). The causality between stock returns and exchange rates: revisited. *Australian Economic Papers, 44(2),* 162-169.

Razzaque, H. B., & Olga, P. (2013). Stock Return and Inflation in Kazakhstan, Russia and Ukraine. *International Journal of Economics and Finance, 5(1),* 253-270.

Smyth, R., & Nandha, M. (2003). Bivariate causality between exchange rates and stock prices in South Asia. *Applied Economics Letters, 10(11),* 699-704.

Soenen, L. A., & Hennigar, E. S. (1988). An analysis of exchange-rates and stock-prices-the united-states experience between 1980 and 1986. *Akron Business and Economic Review, 19(4),* 7-16.

Wu, Y. (2000). Stock prices and exchange rates in VEC model—The case of Singapore in the 1990s. *Journal of Economics and Finance, 24(3),* 260-274.

Zhao, H. (2010). Dynamic relationship between exchange rate and stock price: Evidence from China. *Research in International Business and Finance, 24(2),* 103-112.

Zubair, A. (2013). Causal relationship between stock market index and exchange rate: Evidence from Nigeria. *CBN journal of Applied Statistics, 4(2),* 87-110.